Mathematics – Evolution of Computing

Data Representation In Computers

Did you know that all computers are based on mathematics? Everything you do when you're on a computer, mobile phone or games console is run by numbers. The numbers that a computer uses are '1' and '0' – this is called Binary (or Base-2). So how do two numbers run our digital world?

Humans use a whole range of numbers, and to represent those numbers we use something called Denary (or Base-10). Denary is the number system that we use to represent data and it uses the symbols 0-9, so we can actually represent 10 different values: 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9.

However, computers process and store all of their information (such as text, images, numbers, sounds, programs and instructions) in binary 1's and 0's. With binary you can only represent 2 different values. However, by combining these two numbers together, computers can cleverly represent any number, however long it is! Therefore they're able to store enormous pieces of information and large volumes of data.

Fun Fact: This binary system was created in the 17th Century by a mathematician called Gottfried Leibniz.

If you would like the answers to the activities, please email computerscience@bt.com stating your school and key stage.

Let's start with denary first. With denary we have place values such as: 1,000, 100, 10, 1 and by using the symbols 0-9 we can represent any number value. For example, the number 4,321 is represented by:

Place Values	Symbols (0-9)
1000	4
100	3
10	2
1	1

When we calculate this we do:
1,000 x 4 = 4000
100 x 3 = 300
10 x 2 = 20
1 x 1 = 1

When we add those all up we get
4,000 + 300 + 20 + 1 = 4,321

Now let's move to binary. In binary, the information is stored in bytes. Bytes are made up of 8 place values and the symbols 0 and 1. So for example, 10101011 would be represented by:

Place Values	128	64	32	16	8	4	2	1
Symbol (1 or 0)	1	0	1	0	1	0	1	1

We can now calculate this back into Denary (Base-10).
(128 x 1)+(64 x 0)+(32 x 1)+(16 x 0)+(8 x 1)+(4 x 0)+(2 x 1)+(1 x 1) = 171
or more simply 128+32+8+2+1 = 171

Data Representation In Computers

Extras

Use a pencil so you can reuse this template for pages 5-7 on Activity 1.

Place Values	128	64	32	16	8	4	2	1
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
Symbol (1 or 0)								
....								

Have a Go

- Primary Activities & Games https://atadastral.co.uk/go/bswmh1
- Secondary Problems & Challenges https://atadastral.co.uk/go/bswmh2
- Post-16 - Thinking Mathematically https://atadastral.co.uk/go/bswmh3
- Representation of Numbers https://atadastral.co.uk/go/bswmh4
- Representation of Text https://atadastral.co.uk/go/bswmh5
- Representation of Images https://atadastral.co.uk/go/bswmh6
- Can You Rescue The Diamond? https://atadastral.co.uk/go/bswmh7

Find Out More

- Representing and manipulating data in computers https://atadastral.co.uk/go/bswmf1
- Using numbers and handling data https://atadastral.co.uk/go/bswmf2
- Mathematics for science and technology https://atadastral.co.uk/go/bswmf3
- How Do Scientists Use Maths To Make Their Discoveries? https://atadastral.co.uk/go/bswmf4

Teacher Links

- NRICH aims to enrich the mathematical experiences of all:
  - Primary Curriculum https://atadastral.co.uk/go/bswmt1
  - Secondary Curriculum https://atadastral.co.uk/go/bswmt2
  - Post-16 Curriculum https://atadastral.co.uk/go/bswmt3
- Binary Activities For Primary https://atadastral.co.uk/go/bswmt4
- Hello World issue 10 - Maths and Computer Science Special https://atadastral.co.uk/go/bswmt5

Activity 1
Binary Art

0101010001101
1001101101111 0
0110111101101 1
01110110111101 1

A computer stores everything as 1's and 0's, including images. Use the following activities to see how that is possible.

Look at the grid on the right-hand side, when there is a '1', colour that square in black. When there is a '0', leave it blank.

What image can you see? Now try the same for the two grids below:

0	0	0	1	1	0	0	0
0	0	1	1	1	1	0	1
0	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1
0	0	1	1	1	1	0	1
0	0	0	1	1	0	0	0

0	0	0	1	1	0	0	0
0	0	1	1	1	1	0	0
0	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1
0	1	0	1	1	0	1	0
0	1	1	1	1	1	1	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0

0	0	0	1	0	0	0	0
0	0	1	1	1	0	0	0
0	0	1	0	1	0	0	0
0	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0
0	1	1	0	1	1	0	0
1	1	0	0	0	1	1	0

Activity 1
Binary Art

For these next ones, firstly have a go at converting the binary number in each square into a denary number.

Once you have the denary number, use the key on the right-hand side to colour in the squares in the suggested colour.

What images can you see?

A

00100111	00011001	00100100	00000001	00011111	00010110	00100101
00010111	00011011	00010101	00101010	00011110	00010110	00011001
00011110	00011011	00011000	00011000	00011010	00011101	00011011
00010001	00011011	00011011	00010101	00010101	00011010	00010100
00001110	00000011	00010110	00011100	00011101	00001100	00010001
00001001	00110001	00101111	00011110	00100000	00001111	00100110
00010001	00000111	00010100	00100001	00101101	00011101	00110010

Key:

Denary Range	Colour
21-30	Red

Activity 1

Binary Art

For these next ones, firstly have a go at converting the binary number in each square into a denary number.

Once you have the denary number, use the key on the right-hand side to colour in the squares in the suggested colour.

What images can you see?

B

00011111	00110001	00100001	00000110	00100000	00100011	00101001
00100000	00100110	00010100	00001111	00010010	00101111	00110000
00101111	00001110	00011011	00010011	00010110	00001111	00011111
00001010	00010000	00010010	00001110	00010100	00001011	00000101
00101100	00001100	00101110	00010100	00100111	00010001	00100010
00101000	00001111	00100101	00001100	00100000	00001101	00101011
00100100	00100010	00101100	00101101	00101010	0011111	00110010

Key:

Denary Range	Colour
1-10	Green
11-20	Blue
21-30	Red

Activity 1
Binary Art

For these next ones, firstly have a go at converting the binary number in each square into a denary number.

Once you have the denary number, use the key on the right-hand side to colour in the squares in the suggested colour.

What images can you see?

C

00010010	00001110	00000001	00000010	00000101	00001101	00010100
00001111	00001001	00000111	00000110	00001000	00001001	00001100
00000100	00000101	00101101	00000100	00110010	00001010	00000100
00000100	00000100	00001001	00100111	00000100	00000100	00001000
00001001	00010101	00000011	00000010	00000110	00011110	00000011
00001110	00000101	00011110	00011011	00011000	00000111	00010100
00010000	00010011	00000011	00000101	00000001	00001111	00010010

Key:

Denary Range	Colour
1-10	Green
11-20	Blue
21-30	Red
31-40	Yellow
41-50	Black

Activity 1
Binary Art

For this part of the activity, we've provided you with a blank grid. Have a go at making your own pixel art to challenge your partner with.

D

Key:

Denary Range	Colour
1-10	Green
11-20	Blue
21-30	Red
31-40	Yellow
41-50	Black

Activity 2
Barcodes: Calculating the Check Digit

What is a barcode?

Barcodes are lines (or dots) that can be recognised by machines which help identify what that object is. What items can you see around your room that have barcodes?

8947 0007

The numbers at the bottom of the lines are actually there to show what the lines mean. But what do they mean?

The numbers let machines know who made the item (the manufacturer). But they also give that item a unique number which can't be shared with other products.

Number System Character

Check Digit

9 779362 274510 5

Manufacturer Number

Center Bars

Item Number

The last number is special, and it's called a 'check digit'.

In a shop, the computer has a list of all of the items and their associated barcodes.

When you buy something at a shop, the barcode scanner (connected to a computer) will read the barcode, check it's valid and make a note on the system that the item has been sold.

The problem

When transmitting lots of data, errors can occur, causing some data to be incorrectly received.

For example, if the barcode scanner is moved too quickly it can misread the code and cause data to be stored and transmitted incorrectly.

Machines have special ways of checking if the data being transmitted is correct. In barcodes, we use the last number, the check digit.

9 780072 465211

The last digit of a bar code is special, it is used to make sure the other numbers are correct.

Activity 2

Barcodes: Calculating the Check Digit

How does a computer do this?

The steps a computer follows to calculate the last digit of a 13-digit barcode are:

1. Multiply all the numbers in even positions by 3.
2. Multiply all the numbers in odd positions by 1.
3. Add together the results of Steps 1 and 2.
4. If the units digit (the number furthest on the right) of the result of Step 3 is 0, that is your check digit. If it isn't 0 then you subtract that number from 10 and the result will be your check digit.

Let's try a worked example:

We want to work out the hidden last number (the check digit) behind the pink box.

9 780072 46521

1 Multiply all the numbers in even positions by 3.

Start by putting your numbers into a table.
Then multiply each digit in an even position by 3.

Digit Position	1	2	3	4	5	6	7	8	9	10	11	12	13
Bar code digit	9	7	8	0	0	7	2	4	6	5	2	1	?
Multiplier x		3		3		3		3		3		3	
Answer =		7 x 3 = 21		0 x 3 = 0		7 x 3 = 21		4 x 3 = 12		5 x 3 = 15		1 x 3 = 3	

2 Multiply all the numbers in odd positions by 1.

Digit Position	1	2	3	4	5	6	7	8	9	10	11	12	13
Bar code digit	9	7	8	0	0	7	2	4	6	5	2	1	?
Multiplier x	1	3	1	3	1	3	1	3	1	3	1	3	
Answer =	9 x 1 = 9	7 x 3 = 21	8 x 1 = 8	0 x 3 = 0	0 x 1 = 0	7 x 3 = 21	2 x 1 = 2	4 x 3 = 12	6 x 1 = 6	5 x 3 = 15	2 x 1 = 2	1 x 3 = 3	

3 Add together the results of Steps 1 and 2.

9 + 21 + 8 + 0 + 0 + 21 + 2 +12 +6 15 + 2 + 3 = 99

4 Divide the result of Step 3 by 10.

The units digit of the result of Step 3 is not 0.

So we take the units digit from the result of Step 3 and subtract from 10.
Our units digit is 9 99
10 – 9 = 1
1 is our check digit!

Activity 2
Barcodes: Calculating the Check Digit

Have a go!

Can you work out the missing digit?

5 057753 25512

Digit Position	1	2	3	4	5	6	7	8	9	10	11	12	13
Bar code digit													
Multiplier x													
Answer =													

0 705632 08594

Digit Position	1	2	3	4	5	6	7	8	9	10	11	12	13
Bar code digit													
Multiplier x													
Answer =													

Try to find some items with barcodes that are 13 digits long from the room you're in.

Hide the last digit and see if you can work it out correctly!
Maybe try and set some challenges for your partner?